# IIT JEE PHYSICS

Physics has always been the most interesting subject in IIT JEE and other engineering entrance exams preparation. A poll conducted in 5 IIT's amongst IIT students who cleared IIT JEE clearly indicates that 45% students consider physics as most interesting subject under IIT JEE. Physics is followed by mathematics 35% and then chemistry 20%. What makes Physics so interesting? There are two reasons for it:- 1) The course content itself – it's very practical and can help the reader understand lot of practical day to day events and gadgets 2) The nature of questions asked in IIT JEE physics papers. The questions asked in IIT JEE are always very practical and application based. Unlike IIT JEE chemistry where a direct knowledge of concept can be tested, in physics it's always the understanding of the concept and application of the concept which is tested.

Some topics like – laws of motion in mechanics, modern physics, electrostatics and electric current, waves and sound, some topics in optics (specifically if you know sign convention properly) are highly scoring over others. They concepts involved in them are simpler and questions framed are easier to crack as compared to others like rotational motion, magnetism, simple harmonic motion etc.

The strategy to crack IIT JEE physics exam is to build very good conceptual clarity. Books like Concepts of Physics by H.C. Verma and Resnick Halliday really help to build strong concept and application ability. It's very important to visualize the questions in real life form while trying to figure out solutions for many IIT JEE physics questions. Physics studied at IIT JEE level also helps in engineering in the first year. In fact lot of topics that are studied at JEE level are studied again at advanced level in IIT's. So it's very important to have thorough concept clarity from the beginning in physics. One important caution that has been proved time and again is – not to jump to problem solving without mastering the concept.

## General Physics

In order to be able to answer all scientific inquiries into a universally intelligible format, one has to develop a commonly accepted language in which to converse. It was this need which led to the development of **units and dimensions**. It is an effort to do away with subjectivity of forms and personal prejudices and introduce a common objectivity. If we are to report the result of a **measurement** to someone who wishes to reproduce this measurement, a standard must be defined. Therefore, in order to reduce and eliminate such and other discrepancies, an international committee set up in 1960, established a set of standards for measuring the fundamental quantities.

From the IIT JEE point of view **Measurement, Dimensions, Vectors and Scalars** do not hold lot of significance as we cannot expect a number of questions directly based on this. However, we cannot completely ignore this chapter as this forms the basis of all chapters to follow.

## Dimensions

By international agreement a small number of physical quantities such as length, time etc. are chosen and assigned standards. These quantities are called ‘**base quantities**’ and their units as ‘**base units**’. All other physical quantities are expressed in terms of these ‘base quantities’. The units of these dependent quantities are called ‘**derived units**’.

The standard for a unit should have the following characteristics.

(a) It should be well defined.
(b) It should be invariable (should not change with time)
(c) It should be convenient to use
(d) It should be easily accessible

The 14th general conference on weights and measures (in France) picked seven quantities as base quantities, thereby forming the **International System of Units**abbreviated as SI (System de International) system.

**Base quantities and their units**

The seven base quantities and their units are

| **Base quantity** | **Unit** | **Symbol** |
|---|---|---|
| Length | Metre | M |
| Mass | Kilogram | Kg |
| Time | Second | Sec |
| Electric current | Ampere | A |
| Temperature | Kelvin | K |
| Luminous intensity | Candela | Cd |
| Amount of substance | Mole | Mole |

**Derived units**

We can define all the derived units in terms of base units. For example, speed is defined to be the ratio of distance to time.

Unit of Speed = (unit of distance (length))/(unit of time)

= m/s = $ms^{-1}$ (Read as metre per sec.)

### SOME DERIVED SI UNITS AND THEIR SYMBOLS

| Quantity | Unit | Symbol | Express in base units |
|---|---|---|---|
| Force | newton | N | $Kg\text{-}m/sec^2$ |
| Work | joules | J | $Kg\text{-}m^2/sec^2$ |
| Power | watt | W | $Kg\text{-}m^2/sec^3$ |
| Pressure | pascal | Pa | $Kg\ m^{-1}/S^2$ |

**Important:**

The following conventions are adopted while writing a unit.

**(1)** Even if a unit is named after a person the unit is not written capital letters. i.e. we write joules not Joules.

**(2)** For a unit named after a person the symbol is a capital letter e.g. for joules we write 'J' and the rest of them are in lowercase letters e.g. seconds is written as 's'.

**(3)** The symbols of units do not have plural form i.e. 70 m not 70 ms or 10 N not 10Ns.

**(4)** Not more than one solid's is used i.e. all units of numerator written together before the '/' sign and all in the denominator written after that.
i.e. It is 1 $ms^{-2}$ or 1 $m/s^{-2}$ not 1m/s/s.

**(5)** Punctuation marks are not written after the unit
e.g. 1 litre = 1000 cc not 1000 c.c.

It has to be borne in mind that SI system of units is not the only system of units that is followed all over the world. There are some countries (though they are very few in number) which use different system of units. For example: the FPS (Foot Pound Second) system or the CGS (Centimeter Gram Second) system.

**Dimensions**

The unit of any derived quantity depends upon one or more fundamental units. This dependence can be expressed with the help of dimensions of that derived quantity. In other words, the dimensions of a physical quantity show how its unit is related to the fundamental units.

To express dimensions, each fundamental unit is represented by a capital letter. Thus the unit of length is denoted by L, unit of mass by M. Unit of time by T, unit of electric current by I, unit of temperature by K and unit of luminous intensity by C.

Remember that speed will always remain distance covered per unit of time, whatever is the system of units, so the complex quantity speed can be expressed in terms of length L and time T. Now,we say that dimensional formula of speed is $LT^{-1}$. We can relate the physical quantities to each other (usually we express complex quantities in terms of base quantities) by a system of dimensions.

Dimension of a physical quantity are the powers to which the fundamental quantities must be raised to represent the given physical quantity.

**Applications of Dimensions**

Broadly speaking, dimension is the nature of a Physical quantity. Understanding of this nature helps us in many ways.

Following are some of the applications of the theory of dimensional analysis in Physics:

**(i) To find the unit of a given physical quantity in a given system of units:**

By expressing a physical quantity in terms of basic quantity we find its dimensions. In the dimensional formula replacing M, L, T by the fundamental units of the required system, we get the unit of physical quantity. However, sometimes we assign a specific name to this unit.

**Illustration:**

Force is numerically equal to the product of mass and acceleration

i.e. Force = mass x acceleration

or [F] = mass x velocity/time= mass x displacement/$(time)^2$ ) = mass x

length/$(time)^2$)

$= [M] \times [LT^{-2}] = [MLT^{-2}]$

Its unit in SI system will be Kgms<sup<>-2 which is given a specific name “newton (N)”. Similarly, its unit in CGS system will be $gmcms^{-2}$ which is called “dyne”.

**(ii) To find dimensions of physical constants or coefficients:**

The dimension of a physical quantity is unique because it is the nature of the physical quantity and the nature does not change. If we write any formula or equation incorporating the given physical constant, we can find the dimensions of the required constant or co-efficient.

**Illustration:**

From Newton’s law of Gravitation, the exerted by one mass upon another is

$$F=G\ (m_1\ m_2)/r_2 \text{ or } G=(Fr_2)/(m_1\ m_2\ )$$

$$\text{or } [G] = ([MLT]^2][L^{-2}])\ /\ ([M][M]) = [M^{-1}\ L^3\ T^{-2}\ ]$$

We can find its SI unit which is $m^3/Kgs^2$.

**(iii) To convert a physical quantity from one system of units to another:**

This is based on the fact that for a given physical quantity, magnitude x unit = constant
So, when unit changes, magnitude will also change.

**Illustration:**

Convert one Newton into dyne </sup<>

**Solution:**

Dimensional formula for Newton = $[MLT^{-2}]$

Or 1 N = 1 Kg $m/s^2$ ; But 1 kg = $10^3$ g and 1 m = $10^2$ cm

Therefore 1 N = $((10^3\ g)(10^2\ cm))/s^2 = 10^5\ g\ cm/s^2 = 10^5$ dyne

**(iv) To check the dimensional correctness of a given physical relation:**

This is based on the principle that the dimensions of the terms on both sides on an equation must be same. This is known as the **‘principle of homogeneity’**. If the dimensions of the terms on both sides are same, the equation is dimensionally correct, otherwise not.

**Caution:** It is not necessary that a dimensionally correct equation is also physically correct but a physically correct equation has to be dimensionally correct.

**Illustration:**

(i) Consider the formula, $T=2\pi\sqrt{(l/g)}$

Where T is the time period of oscillation of a simple pendulum in a simple harmonic motion, l and g are the length of the pendulum and gravitational constants respectively. Check this formula, whether it is correct or not, using the concept of dimension.

As we know $[g] = [LT^2]$

Therefore $[T] = \sqrt{([L])/([LT^{-2}])} = [T]$ s

Thus the above equation is dimensionally correct (homogenous) and later you will come to know that it is physically also correct.

(ii) Consider the formula $s=ut -1/3\ at^2$. Check this formula whether it is correct or not, using the concept of dimension.

Dimensionally

$$[L] = [LT^{-1}]\ [L] – [LT^{-2}]\ [T^2]$$

$$=> [L] = [L] – [L]$$

In this case also the formula is dimensionally correct but, you know that it is physically incorrect as the correct formula is given by

$$S = ut + 1/3at^2$$

**(v) As a research tool to derive new relations:**

One of the aims of scientific research is to discover new laws relating different physical quantities. The theory of dimensions (in the light of principal of homogeneity) provides us with a powerful tool of research in the preliminary stages of investigation [It must be again emphasized that mere dimensional correctness of an equation does not ensure its physical correctness]

**Limitations of the theory of dimensions**

The limitations are as follows:

**(i)** If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimension. For example, if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be work or energy or even moment of force.

**(ii)** Numerical constants, having no dimensions, cannot be deduced by using the concepts of dimensions.

**(iii)** The method of dimensions cannot be used to derive relations other than product of power functions. Again, expressions containing trigonometric or logarithmic functions also cannot be derived using dimensional analysis, e.g.

$$s = ut + 1/3at^2 \quad \text{or} \quad y = a\ \text{sincot} \quad \text{or} \quad P= P_0e\ (–Mgh)/RT$$

cannot be derived. However, their dimensional correctness can be verified.

**(iv)** If a physical quantity depends on more than three physical quantities, method of dimensions cannot be used to derive its formula. For such equations, only the dimensional correctness can be checked. For example, the time period of a physical pendulum of moment of inertia I, mass m and length l is given by the following equation.

$T = 2\pi\sqrt{(I/mgl)}$ (I is known as the moment of Inertia with dimensions of $[ML^2]$ through dimensional analysis), though we can still check the dimensional correctness of the equation (Try to check it as an exercise).

**(v)** Even if a physical quantity depends on three Physical quantities, out of which two have the same dimensions, the formula cannot be derived by theory of dimensions, and only its correctness can be checked e.g. we cannot derive the equation.

## Scalars and Vectors

### Scalars

Physical quantities which can be completely specified by a **number and unit**, and therefore have the **magnitude only**, are scalars. Some physical quantities which are scalar are mass, length, time, energy etc. These examples obey the algebraic law of addition.

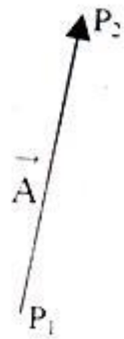
### Vectors

Vectors are physical quantities, which besides **having both magnitude and direction also obey the law of geometrical addition.** (The law of geometrical addition, i.e. the law of triangular addition and law of parallelogram are discussed later in this chapter). Some physical quantities, which are vectors, are displacement, velocity, force etc.

### Representation of a Vector

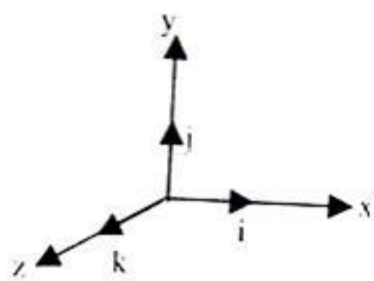
Since vectors have directions, any representation of them has to include the direction.

To represent a vector we use a line with an arrow head. The length of the line represents the magnitude of vector and direction of the arrow represents the direction of the vector. Let us start with a vector quantity called **displacement.** In the enclosed figure the change of position from point $P_1$ to $P_2$ is represented graphically by the directed line segment with an arrowhead to represent direction of motion.




Vector is a Physical quantity and all physical quantities have units. Hence, the vectors also have units, they are called unit vectors.

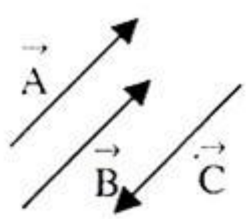
**Unit Vectors:** A unit vector is a vector having a **magnitude of unity.** Its only purpose is to describe a direction in space. On x-y co-ordinate system ĩ denote unit vector in positive x direction and ĵ denotes unit vector in positive y direction.




Any vector in x – y plane can be represented in terms of these unit vectors ĩ & ĵ.

Similarly any vector in a 3 dimensional x y z space can be represented in terms of unit vectors ĩ, ĵ and k where, k is the unit vector in the positive z direction, as shown in figure above.

**Parallel Vectors:** Two or more vectors are said to be parallel when they are parallel to the same line. In the figure below, the vectors A B and C are all parallel.




**Equal Vectors:** Two or more, vectors are equal if they have the same magnitude (length) and direction, whatever their initial points. In the figure above, the vectors A and B are equal.

**Negative Vectors:** Two vectors which have same magnitude (length) but their direction is opposite to each, other called the negative vectors of each other. In figure above vectors A and C or B and C are negative vectors.

**Null Vectors:** A vector having zero magnitude is called zero **vector or 'null vector'** and is written as = O vector. The initial point and the end point of such a vector coincide so that its direction is indeterminate.

The concept of null vector is hypothetical but we introduce it only to explain some mathematical results.

**Invariance of the vector:** Any vector is invariant so it can be taken anywhere in the space keeping its **magnitude and direction same**. In other words, the vectors remain invariant under translation.

## Multiplication of Vectors

### 1. Multiplication of vector by a scalar

Let vector a is multiplied by a scalar m. If m is a positive quantity, only magnitude of the vector will change by a factor 'm' and its direction will remain same. If m is a negative quantity the direction of the vector will be reversed.

### 2. Multiplication of a vector by a vector

(i) Dot product or scalar product
(ii) Cross product or vector product

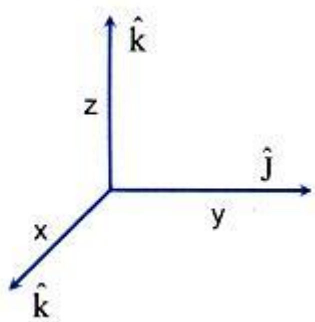
### Dot product or scalar product

The dot product of two vectors a and b is defined as

$$a^{->} . b^{->} = ab \cos\theta$$

where a and b are the magnitudes of the respective vectors and θ is the angle between them. The final product is a scalar quantity. If two vectors are mutually perpendicular then $\theta = 90^0$ and cos 90 = 0, Hence, their dot product is zero.

Some examples of dot product: work = $\vec{F} \cdot \vec{s}$ = Fs cosθ




Here,

$$\hat{\imath}.\hat{\jmath} = 0,\ \hat{\imath}.\hat{k} = 0, \&\ \hat{\jmath}.\hat{k} = 0 \text{ and } \hat{\imath}.\hat{\imath} = 1,\ \hat{\imath}.\hat{\jmath} = 1,\ \&\ \hat{k}.\hat{k} = 1$$

The dot product obeys commutative law

i.e. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

Hence, $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

**Illustration :**

Find the angle between the vectors A and B where

$$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}, \vec{B} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

**Solution :**

We know

$\vec{A}.\vec{B}$ = |A||B| cosθ where |A| = $\sqrt{(2^2 + 3^2 + 3^2)} = \sqrt{22}$,

$|B| = \sqrt{(1^2 + 2^2 + 3^2)} = \sqrt{14}$

Hence $\cos\theta = (\vec{A} . \vec{B})/(|A||B|) = ((2i + 3j + 3k)(i + 2j - 3k))/(\sqrt{22} \times \sqrt{14})$

$= (2+6-9)/(2\sqrt{77}) = (-1)/(2\sqrt{77})$

$\Rightarrow \theta = \cos^{-1}((-1)/(2\sqrt{77}))$

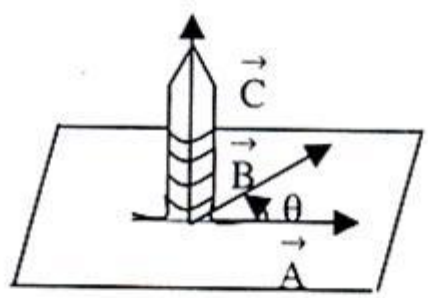
**Cross product or vector product**

The cross product of the two vectors a and b is defined as

$\vec{a} \times \vec{b} = \vec{c}$

Here, $|\vec{c}| = |\vec{a}| \times |\vec{b}| \sin\theta$, where θ is the angle between the vectors.

Vector product is defined as a vector quantity with a direction perpendicular to the plane containing vectors A and B then C = AB sin θ n, where n is a unit vector perpendicular to the plane of vector A and vector B. To specify the sense of the vector C, refer to the figure given below.




Imagine rotating a right hand screw whose axis is perpendicular to the plane formed by vectors A and B so as to turn it from vectors A to B trough the angle θ between them. Then the direction of advancement of the screw gives the direction of the vector product vectors $\vec{A} \times \vec{B}$.

**Illustration:**

Obtain a unit vector perpendicular to the two vectors $\vec{A} = 2i + 3j + 3k$, $\vec{B} = i - 2j + 3k$

**Solution:**

We know that A-> × B-> = AB sin θ n

n = (A->×B->)/ABsinθ

We have A-> × B-> = 17i-2j-7k

A = √29 B = √14

and θ = $\cos^{-1}$ 8/(√14 √29) (Use concept of dot product to find θ).

From the above values we can find n

Solving we get,

n = (17i-2j-7k)/(√29 √14 sinθ) where θ $\cos^{-1}$ 8/(√14 √29)

**Cross Product of Parallel vectors**

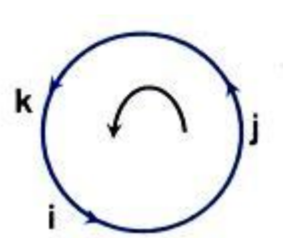
If two vectors are parallel or antiparallel, then θ is either $0^0$ or $180^0$. Since sin $0^0$and sin $180^0$ both equals zero. Hence magnitude of their cross product is zero.

The vector product does not follow commutative law.

$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

**Product of unit vectors**




$$\hat{\imath} \times \hat{\jmath} = \hat{k}, \quad \hat{\jmath} \times \hat{\imath} = -\hat{k}, \quad \hat{\jmath} \times \hat{k} = \hat{\imath}, \quad \hat{k} \times \hat{\jmath} = -\hat{\imath}, \quad \hat{k} \times \hat{\imath} = \hat{\jmath}, \quad \hat{\imath} \times \hat{k} - \hat{\jmath}$$

$$\hat{\imath} \times \hat{\imath} = 0, \quad \hat{\jmath} \times \hat{\jmath} = 0, \quad \hat{k} \times \hat{k} = 0 \; and \; \hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \quad \hat{k} \times \hat{\imath} = \hat{\jmath}$$

Multiplication of vectors is a very important topic from IIT JEE, AIEEE and other engineering exams perspective. Dot product or scalar product and cross product or vector product find their use both in physics and mathematics while preparing for IIT JEE, AIEEE and other engineering entrance exams.

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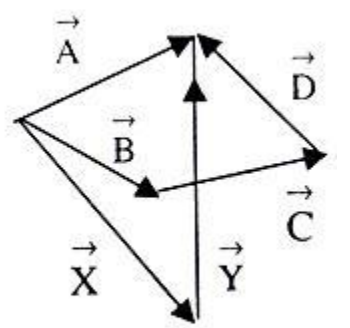
**Vector Components**

**Components of a Vector:**

From the figure given below we can write

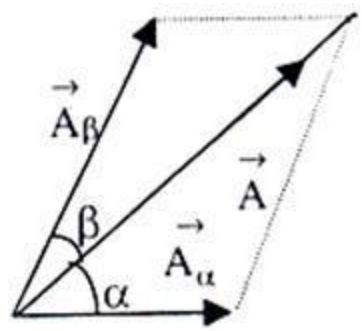
$$\vec{A} = \vec{B} + \vec{C} + \vec{D}.$$

It means B ,C and D vectors are the components of vector A




Note that we can also write A =X +Y. So vectors X and Y are also components of vector A. It implies that we can draw any number of set of components in any desired direction.

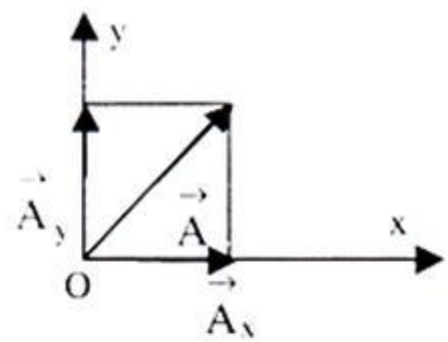
For resolving a vector A along two directions making angles α and with it as shown in figure given below, we use the following:-




Aα = (A sinβ)/(sin(α + β))

**Perpendicular Components**

Representation of any vector lying in the x – y plane, as shown in figure given below, as the sum of two vectors, one parallel to the x-axis and the other parallel to y-axis, is extremely useful in physical analysis because both have mutually independent effects. These two vectors are labeled A and A. These are called the**perpendicular or rectangular component** of vector A and are expressed as:




$$\vec{A} - \vec{A}_x + \vec{A}_y$$

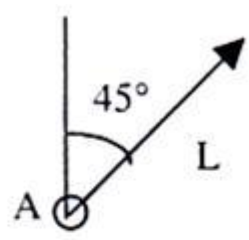
If magnitude and direction of vector A are known then $A_x$ = A cosθ and $A_y$ = A sinθ. Hence we can write

$$A_x^2 + A_y^2 = A_2 \text{ and } \tan\theta = A_y/A_x$$

Now, it should be clear that if we have to add 30 vectors, we will resolve each of these 30 vectors in rectangular components in any x and y direction. Then simply add all the components in the x direction and all the components in the y direction. Adding these two resultant perpendicular components will give us the final resultant.

**Illustration:**

A vector quantity of magnitude L acts on a point A along the direction making an angle of $45^0$ with the vertical, as shown in the figure given below. Find the component of this vector in the vertical direction?




**Solution:**

The component of the vector in the vertical direction will be

$$L*(\cos \pi/4) = L/\sqrt{2}.$$

Learning **components of a vector** is very important from **IIT JEE, AIEEE and other engineering exams perspective**. Most of the Physics is based on usage of vectors and components of vectors. Vectors and their components are also useful in trigonometry and mathematics as a whole. This is a very useful concept and should be learned thoroughly at the beginner's level so that it paves a way for great understanding of statistics, physics and mathematics at higher levels.



Mechanics

## Introduction to Motion in One Dimension

### A body

A certain amount of matter limited in all directions and consequently having a finite size, shape and occupying some definite space is called a body.

### Particle

A particle is defined as a portion of matter infinitesimally small in size so that for the purpose of investigation, the distance between its different parts may be neglected. Thus, a particle has only a definite position, but no dimension. In the problems we are going to discuss, we will consider a body to be a particle for the sake of simplicity.

### MOTION IN ONE DIMENSION

### Motion

The position of object can change on a straight line (like on x-axis with respect to origin) or on a plane with respect to some fixed point on frame. So we can define motion as follows:-

An object or a body is said to be in motion if its position continuously changes with time with reference to a fixed point (or fixed frame of reference).

**Caution :** The moving object is either a particle, a point object (such as an electron) or an object that moves like a particle. A body is said to be moving like if every portion of it moves in the same direction and at the same rate.

### Motion in One Dimension

When the position of object changes on a straight line i.e. motion of object along straight line is called motion in one dimension.

To understand the essential concepts of one dimensional motion we have to go through some basic definitions.

### Frame of reference

One can see the platform from a running train, and it seems that all the objects placed on platform are continuously changing their position. But one, who is on platform, concludes that the objects on the platform are at rest. It means if we will take the trains are reference frame the objects are not stationary and taking reference frame as platform the objects are stationary. So the study of motion is a combined property of the object under study and the observer. Hence there is a need to define a frame of reference under which we have to study the motion of an object.

**Definition**

A frame of reference is a set of coordinate axes which is fixed with respect to a space point (a body or an object can also be treated as a point mass therefore it can become a site for fixing a reference frame), which we have arbitrarily chosen as per our observer's requirement. The essential requirement for a frame of reference is that, it should be rigid.

**Position of an object**

The position of an object is defined with respect to some frame of reference. As a convention, we define position of a point (essentially we treat body as a point mass) with the help of three co-ordinates X, Y and Z. Hence X, Y, Z is a set of coordinate axes representing a 3-dimensional space and each point in this space can be uniquely defined with the help of a set of X, Y and Z coordinate, all three axes being mutually perpendicular to each other. The line drawn from origin to the point represents the position vector of that point.

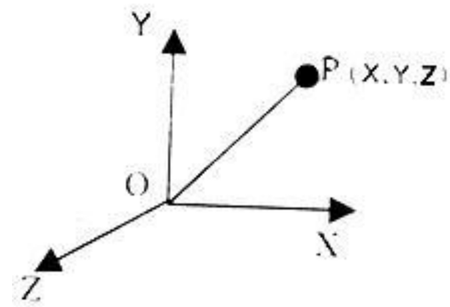
## Introduction to Motion in One Dimension

**Position vector**

It describes the instantaneous position of a particle with respect to the chosen frame of reference. It is a vector joining the origin to the particle. If at any time, (x, y, z) be the Cartesian coordinates of the particle then its position vector is given by vector $\vec{r}$ = xi + yj + zk.

In one-dimensional motion: vector $\vec{r}$ = xi,     y = z = 0   (along x-axis)

In two-dimensional motion: vector $\vec{r}$ = xi + yj               (in x-y plane z = 0)

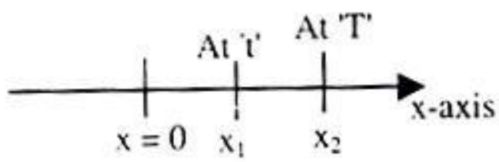



In the figure above, the position of a point P is specified and vector OP is called the position vector.

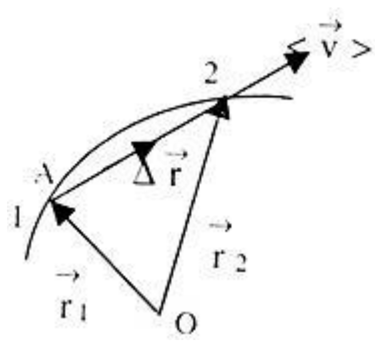
**Displacement**

Consider a case in which the position of an object changes with time. Suppose at certain instant 't' the position of an object is $x_1$ along the x axis and some other instant 'T' the position is $x_1$ then the displacement Δx is defined as

$$\Delta x = x_2 - x_1$$




It can be seen in the figure above where $x_1$ and $x_2$ are instantaneous position of the object at that time.

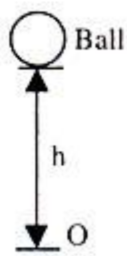



Now consider the motion of a point A with respect to a reference point O. The motion of point A makes its radius vector vary in the general case both in magnitude and in direction as shown in figure above. Suppose the point A travels from point 1 to point 2 in the time interval Δt. It is seen from the figure that the displacement vector $\Delta \vec{r}$ of the point A represents the increment of vector $\vec{r}$ in time Δt:

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_t$$

**Difference between distance and displacement**

To understand the difference between distance and displacement, we study the motion of vertical throw of a ball with respect to point O, as shown in the figure below, to height h.




After some time it will come again to the same point O. The displacement of ball is zero but there is some distance traversed by the ball. It's because distance is a scalar quantity but displacement is a vector quantity.

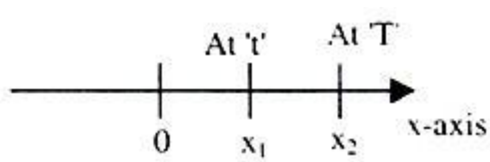
**Uniform and Non Uniform Motion**

Speed is the rate of change of distance without regard to directions. Velocity is the rate at which the position vector of a particle changes with time. Velocity is a vector quantity whereas speed is scalar quantity but both are measured in the same unit m/sec.

The motion of an object may be uniform or non-uniform depending upon its speed. In case of uniform motion the speed is constant, whereas in the non-uniform motion, the speed is variable.

In uniform motion in one dimension the velocity (v) is mathematically defined as

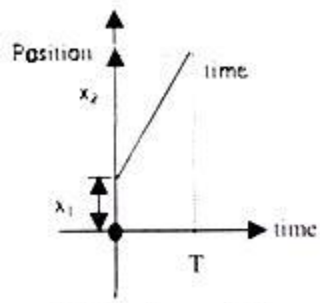
$$v = (x_2 - x_1)/(T-t) \quad \text{...... (1)}$$



where $x_1$ and $x_2$ are instantaneous displacement as shown in figure above at time 't' and 'T' respectively.

**Graphical representation of the uniform motion**

Form the equation (1) we have the following equation




Position time graph for uniform motion

$$x_2 = x_1 + v(T - t)$$

where v is constant. Take t = 0, the equation becomes $x_2 = x_1 + vT$, from this equation it follows that the graph of position of object '$x_2$' against 'T' is a straight line, cutting off $x_1$ on the position axis where $x_1$ is the distance of the particle from the origin at time t = 0.

v = slope of the graph which is constant

**Velocity Vector in Non Uniform Motion**

In any non-uniform motion, we can define an average velocity over a time interval. Average velocity $\vec{v}$ is the ratio of the displacement Δx (that occurs during a particle time interval Δt) to that interval of time i.e.

$$< \vec{v} > = \frac{\Delta \vec{x}}{\Delta t}$$

Now refer to the example, related to figure 2.3, the ratio of $\Delta \vec{r}/\Delta t$ is called the average velocity $<\vec{v}>$ during the time interval Δt. The direction of the vector $<\vec{v}>$ coincides with that of $\Delta \vec{r}$. Average velocity is also a vector quantity.

If at any time $t_1$ position vector of the particle is $\vec{r}_1$ and at time $\vec{r}_2$ position vector is $\vec{r}_2$ then for this interval $\vec{v}_{avg} = \frac{\vec{r}_2 - \vec{r}_1}{\vec{t}_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$

**Instantaneous velocity**

Instantaneous velocity is defined as the rate of change of displacement.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \Rightarrow \vec{v}\ \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

**Illustration:**

This question contains statement-1 (Assertion) and Statement-2 (Reason). Question has 4 choices (A), (B), (C) and (D) out of which only one is correct.

**Statement-1**

A bus moving due north take a turn and starts moving towards east with same speed. There will be no change in the velocity of the bus.

**Statement-2**

Velocity is a vector quantity.

(A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is true, Statement-2 is false.

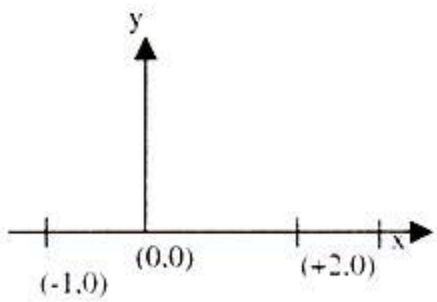
(D) Statement-1 is false, Statement-2 is true.

**Solution (D)**

This is so because bus is changing its direction of motion.

**Illustration:**

A man started running form origin and went up to (2, 0) and returned back to (-1, 0) as shown in figure 2.7. In this process total time taken by man is 2 seconds. Find the average velocity and average speed.



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**Introduction to Motion in One Dimension**

**Solution:**

The man is displaced form origin to (-1, 0)

Hence displacement, s = $\hat{i}$

So average velocity = Displacement/total time = $\hat{i}$/2 = (-1/2) $\hat{i}$ m/sec.

where as, since the total distance traveled by man

= (0, 0) to (2, 0)+(2, 0) to (0, 0)+(0, 0) to (-1, 0)

= 2 + 2 + 1 = 5 m

Hence average speed

= (Total distance)/(Total time)=5/2 m/sec.

**Velocity**

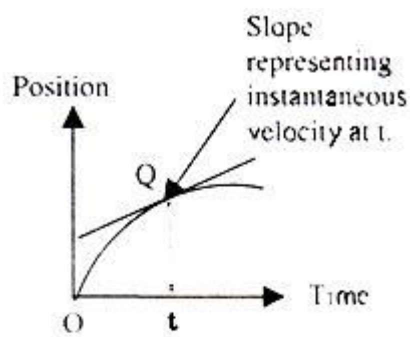
The velocity at any instant is obtained from the average velocity shrinking the time interval closer to zero. As Δt tends to zero, the average velocity approaches a limiting value, which is the velocity at that instant, called instantaneous velocity, which is a vector quantity, mathematically we can define it as

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

The magnitude v of the instantaneous velocity is called the speed and is simple the absolute value of $\vec{v}$ i.e. $|\vec{v}| = \left|\frac{d\vec{x}}{dt}\right|$

In the example related with figure given below, the instantaneous velocity is

$$\vec{v} = \frac{d\vec{x}}{dt}$$




Hence instantaneous velocity is the rate at which a particle's position is changing with respect to time at a given instant. The velocity of a particle at any instant is the slope (tangent) of its position curve at the point representing that instant of time, as shown in figure above.

**Speed**

Speed is defined as rate of change of distance with time.

In any interval of time, average speed is defined as

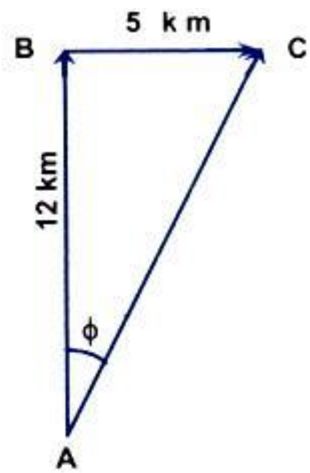
<speed> = (total distance)/(total time taken) = Δs/Δt. As Δs ≥ |Δ $\vec{r}$ |, hence
<speed> ≥ <velocity>

**Illustration:**

A cyclist moves 12 km due to north and then 5 km due east in 3 hr. Find (a) his average speed, (b) average velocity, in m/s.

**Solution:**

In the figure, A shows the initial position and C the final position of the cyclist. The total distance covered by the cyclist AB+BC= (12+5)km = 17 km.




∴ Its average speed = 17/3 km/hr = 1.57 m/s

Its displacement is AC and the magnitude is given by

$AC = \sqrt{(AB^2 + BC^2)} = \sqrt{(12^2 + 5^2)}$ km = 13 km

∴ Its average velocity = 13/3 km/hr

= 1.2 m/s along AC, i.e at $\tan^{-1}(5/12)$ or $22.6^o$ East of North.

**Illustration:**

A train is moving with a constant speed of 5 m/s and there are two persons A and B standing at a separation of 10 m inside the train. Another person C is standing on the ground. Then, find

(a) displacement covered by A, if he moves towards B and back to its position in 10 seconds in frame of reference of train and in frame of reference of C.

(b) distance covered by A in frame of reference of train and in frame of reference of C.

**Solution:**

(a) In the frame of train, displacement covered by A is zero and in frame of reference of C, displacement covered by A=0 + 5x10 = 50 m.

(b) Distance covered by A in frame of reference of train is 20m and distance covered by A in frame of reference of C is (20 + 50) = 70 m.

**Acceleration**

Acceleration is the rate of change of velocity with time. The concept of acceleration is understood in non-uniform motion. It is a vector quantity.

Average acceleration is the change in velocity per unit time over an interval of time.

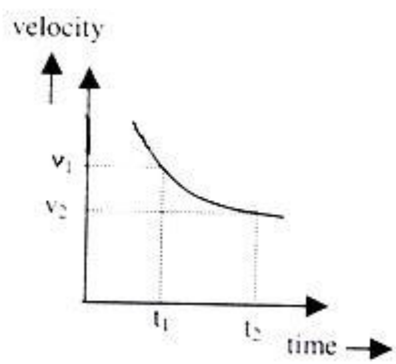
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta\vec{t}} = \frac{\vec{v}_2 - \vec{v}_1}{\vec{t}_2 - \vec{t}_1}$$

Instantaneous acceleration is defined as

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\Rightarrow \vec{a}\ \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

**Acceleration vector in non uniform motion**



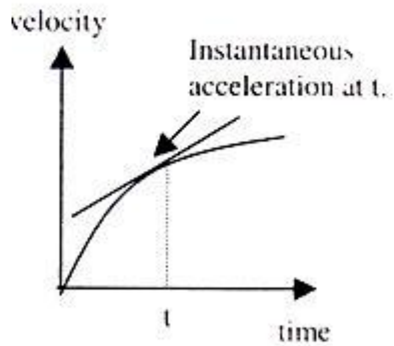

Suppose that at the instant $t_1$ a particle as in figure above, has velocity $\vec{v}_1$and at $t_2$, velocity is $\vec{v}_2$. The average acceleration < $\vec{a}$ > during the motion is defined as

$$< \vec{a} > = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

**Variable Acceleration**

The acceleration at any instant is obtained from the average acceleration by shrinking the time interval closer zero. As Δt tends to zero average acceleration approaching a limiting value, which is the acceleration at that instant called instantaneous acceleration which is vector quantity.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$




i.e. the instantaneous acceleration is the derivative of velocity.

Hence instantaneous acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Instantaneous acceleration at any point is the slope of the curve v (t) at that point as shown in figure above.

**Equations of motion**

The relationship among different parameter like displacement velocity, acceleration can be derived using the concept of average acceleration and concept of average acceleration and instantaneous acceleration.

When acceleration is constant, a distinction between average acceleration and instantaneous acceleration loses its meaning, so we can write

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t - t_0} = \frac{d\vec{v}}{dt}$$

where $\vec{v}_0$ is the velocity at t = 0 and $\vec{v}$ is the velocity at some time t

Now $\vec{a}\ t = \vec{v} - \vec{v}_0$

Hence, $\vec{v} - \vec{v}_0 = \vec{a}\ t$ ................. (2)

This is the first useful equation of motion.

Similarly for displacement

$$\vec{x} = \vec{x}_0 + <\vec{v}> t \qquad \text{................. (3)}$$

in which $\vec{x}_0$ is the position of the particle at $t_0$ and < $\vec{v}$ > is the average velocity between $t_0$ and later time t. If at $t_0$ and t the velocity of particle is

$$<\vec{v}> = ½ (\vec{v}_0 + \vec{v})$$

$$= ½ [\vec{v}_0 + \vec{v}_0 + \vec{a}\ t]$$

$$<\vec{v}> = \vec{v}_0 + \vec{a}\ t/2 \qquad \text{..................... (4)}$$

From equation (3) and (4), we get

$$\vec{x} - \vec{x}_0 = \vec{v}_0\, t + \frac{1}{2}\vec{a}\ t^2 \qquad \text{..................... (5)}$$

This is the second important equation of motion.

Now from equation (2), square both side of this equation we get

$$v_2 = v_0^2 + a^2t^2 + 2v_0\, \vec{a}\ t = v_0^2 + 2\, \vec{a}\ t + [v_0 + \vec{a}\ t/2]$$

$$= v_0^2 + 2\, \vec{a}\ t < v > \text{(Use equation 4)}$$

Use equation (3), to get

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0) \qquad \text{........................ (6)}$$

This is another important equation of motion.